

CBSE CLASS 9 MATHEMATICS

NCERT Curriculum Study Series — 2026 Edition

CHAPTER 11: CIRCLES

ULTRA-PREMIUM WEBSITE-READY STUDY NOTES

Complete Theoretical Notes, Formal NCERT Theorems, Visual Flowcharts, and Quick Revision Tools



KEY STRUCTURAL COMPONENTS INSIDE:

- **Theoretical Frameworks:** Comprehensive definitions mapping the locus of points, boundaries, chords, sectors, segments, and angular subtensions.
- **Rigorous Theorem Analysis:** Detailed tracking of all six mandatory NCERT circle properties with complete geometric justifications.
- **Symmetry & Flowcharts:** Structural mapping illustrating internal relational paths from definitions to proven line theorems.
- **Revision Toolkits:** High-yield comparative analysis matrices paired with diagnostic exam mistake-prevention guides.

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1. CHAPTER INTRODUCTION

In our exploration of geometry, we have primarily focused on shapes built from straight line segments, such as triangles and quadrilaterals. A **Circle** introduces us to the geometry of curved spaces. A circle is one of the most symmetrical and perfect shapes in existence. It represents a continuous path where every single boundary point is perfectly equidistant from a fixed interior point.

Historically, the discovery of the circle and the development of the wheel completely transformed human civilization. In mathematics, circles form the absolute framework for rotational mechanics, trigonometry, astronomy, and calculus. This chapter introduces the essential vocabulary of chords, sectors, and segments, establishing the core theorems that govern circular properties in secondary-level curriculum.



REAL-LIFE APPLICATIONS OF CIRCULAR GEOMETRY

- **Mechanical Engineering:** Gears, wheels, pulleys, and engine pistons rely completely on circular symmetry to convert translational energy into rotational power without friction-based wear.
- **Satellite Navigation & Tracing:** Radar horizons, cellular tower coverage ripples, and satellite orbital paths are plotted using circular coordinate tracking equations.

2. DETAILED EXPLANATION OF CONCEPTS

Introduction to the Circle

Mathematically, a **Circle** is defined as the collection of all points in a plane which are at a fixed, constant distance from a fixed point in that plane. The fixed point is called the **centre**, and the constant distance is called the **radius**.

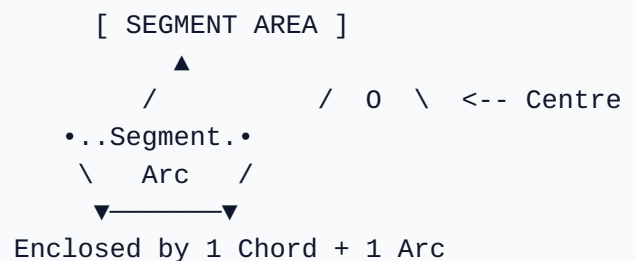
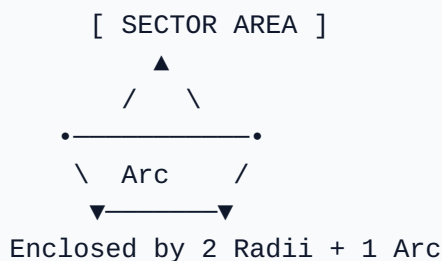
[Image of a basic circle structure with centre O and radius r]

Anatomy & Structural Parts of a Circle

To analyze circular theorems, we must break down a circle into its component parts:

- **Centre (O):** The fixed internal point from which all boundary distances are measured. It does not lie on the circle itself, but inside the region it encloses.
- **Radius (r):** The straight line segment connecting the centre to any point on the boundary line. All radii of a given circle are equal.
- **Diameter (d):** A straight chord passing directly through the centre of the circle. It is the longest chord of the circle and is exactly twice the length of the radius ($d = 2r$).
- **Circumference:** The total linear length of the outer boundary of the circle. It represents its perimeter.
- **Chord:** A straight line segment connecting any two distinct points on the circle's boundary.
- **Arc:** A continuous piece or portion of the circumference of a circle. If the arc is less than a semi-circle, it is a **Minor Arc**; if it is greater, it is a **Major Arc**.
- **Segment:** The region enclosed between a chord and either of its arcs. A chord divides a circle into a **Minor Segment** and a **Major Segment**.
- **Sector:** The region enclosed between an arc and the two radii connecting its endpoints to the centre. Think of it as a slice of pizza. It is divided into a **Minor Sector** and a **Major Sector**.
- **Semi-circle:** A diameter divides a circle into two equal arcs. Each of these equal portions is a semi-circle.

TEXT-BASED VISUALIZATIONS OF SECTOR AND SEGMENT:



Chords and Angles Subtended at the Centre

When you draw a chord and connect both of its endpoints to the centre of the circle using radii, the angle formed between those two radii is called the **Angle Subtended by the Chord at the Centre** (also known as the Central Angle).

[Image showing an angle subtended by a chord at the centre of a circle]

Theorem 11.1: Equal Chords Angle Theorem

Statement: Equal chords of a circle subtend equal angles at the centre.

Deductive Proof Logic: Suppose AB and CD are two equal chords of a circle with centre O . In triangles $\triangle AOB$ and $\triangle COD$, $OA = OC$ (radii), $OB = OD$ (radii), and $AB = CD$ (given). Therefore, $\triangle AOB \cong \triangle COD$ by the SSS congruence criterion. By CPCT, we find $\angle AOB = \angle COD$, proving the theorem.

Theorem 11.2: Converse of Equal Chords Theorem

Statement: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Perpendicular from Centre to a Chord

Theorem 11.3: Perpendicular Bisection Theorem

Statement: The perpendicular from the centre of a circle to a chord bisects the chord.

Geometrical Value: If a line is drawn from centre O perpendicular to a chord AB meeting it at point M ($\angle OMA = 90^\circ$), then M is the midpoint of AB , meaning $AM = MB$. This theorem allows us to use Pythagoras Theorem ($\triangle OMA$ is a right triangle) to calculate radii or chord distances easily.

[Image of the perpendicular from centre to chord showing a right-angled triangle construction]

Theorem 11.4: Converse of Perpendicular Bisection Theorem

Statement: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Equal Chords and Their Distances from the Centre

The distance of a line from a point is defined as the length of the perpendicular segment drawn from that point to the line. Using this definition, circles exhibit deep internal symmetry:

- **Theorem 11.5:** Equal chords of a circle (or of congruent circles) are equidistant from the centre.
- **Theorem 11.6 (Converse):** Chords equidistant from the centre of a circle are equal in length.

Circle Symmetry Properties

A circle possesses an infinite amount of symmetry. Any straight line passing through its centre forms a **line of symmetry**, meaning a circle has infinitely many lines of symmetry. Rotational symmetry is also infinite; rotating a circle by any angle around its centre leaves its appearance completely unchanged.

3. IMPORTANT TERMS & DEFINITIONS

TERMINOLOGY	MATHEMATICAL DEFINITION	STUDENT-FRIENDLY EXPLANATION
Circle	The locus of all points in a plane equidistant from a fixed interior point.	A perfectly round flat shape whose outer edge is the same distance from the center all around.
Chord	A straight line segment whose endpoints both lie on the boundary of a circle.	Any straight line cut across a circle connecting two points on its edge.
Arc	A continuous path segment forming a portion of a circle's circumference.	A curved section of the outer edge of a circle.
Sector	A circular region bounded by a minor or major arc and two bounding radii.	A pie-slice shape cut out of a circle by two radius lines.
Segment	A circular region bounded by a straight chord line and an intersecting arc.	A slice cut off from a circle by a straight chord line.
Central Angle	The angle subtended at the centre of a circle by an arc or chord.	The corner angle formed at the center of the circle by two radius arms.
Perpendicular Bisector	A line intersecting another segment at 90° that cuts it into two equal halves.	A crossing line that forms a perfect T-square and splits a line exactly in half.

4. NCERT KEY POINTS

- **The Longest Chord Law:** The diameter is the **longest chord** of any circle. Its length is equal to twice the radius ($d = 2r$).
- **The Chords Proximity Rule:** As a chord moves closer to the centre of a circle, its length grows longer. The chord reaches its maximum length when it passes through the centre, where its distance from the centre is zero.
- **Three-Point Constraint:** While an infinite number of circles can pass through one or two points, there is **one and only one unique circle** that can pass through three distinct non-collinear points.

RIGOROUS GEOMETRIC PROPERTY NOTE

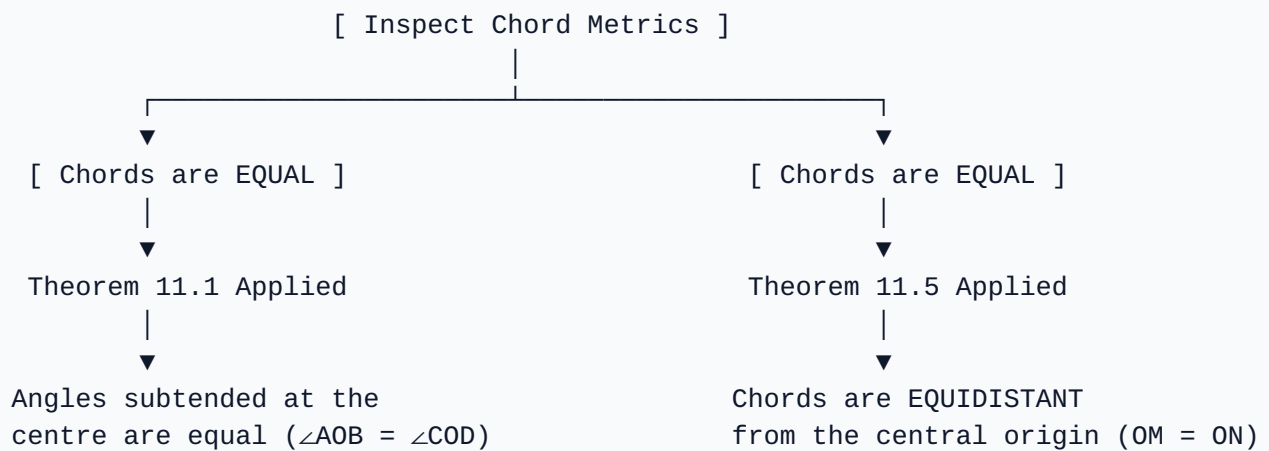
If you connect a point to a chord using a perpendicular line, that line represents the **shortest distance** from the point to the chord. In a circle, this perpendicular line always aligns perfectly with the centre.

5. TABLES & LOGIC FLOWCHARTS

Circular Component Comparison Matrices

FEATURE COMPARED	CIRCULAR SECTOR REGION	CIRCULAR SEGMENT REGION
Enclosing Boundaries	Enclosed by 2 Radii and 1 Arc.	Enclosed by 1 Chord and 1 Arc.
Center Reference	The vertex point always connects directly to the centre.	The boundary chord can bypass the center entirely.
Visual Metaphor	A wedge slice of pizza or cake.	A flat cap slice cut off a round loaf of bread.

CHORD LOGIC RELATIONSHIPS FLOWCHART:



6. MEMORY TRICKS (*JALDI YAAD KARNE KE TAREEKE*)

1. Sector vs. Segment Spelling Anchor

Students frequently confuse sectors and segments. Use this vocabulary trick:

- **SeCtor** contains the letter **C**, which stands for **C**entre. A sector must connect directly to the **C**entre using radius arms.
- **SegMent** contains the letter **M**, which stands for **M**iddle chord line. A segment is bounded by a flat chord line cut through the circle.

2. Perpendicular Bisection Action phrase

Remember this simple rhyme for chords: "From the centre, drop a square (90°), and you split the chord right there (bisect)." This reminds you that a perpendicular line from the centre always cuts a chord into two equal halves.

7. COMMON MISTAKES STUDENTS MAKE

ERROR 1: CONFUSING THE RADIUS WITH THE DIAMETER IN EQUATIONS

The Mistake: Plugging the diameter length directly into formulas that require the radius variable (r), leading to calculations that are double the correct value.

How to avoid it: Always check your values first. If a problem states "a circle has a diameter of 10 cm," immediately calculate the radius by dividing by two: $r = 10 / 2 = 5$ cm before starting your geometric calculations.

ERROR 2: MIXING UP EQUAL ANGLES WITH EQUAL DISTANCES

The Mistake: Stating that equal chords are equal to their central angle measures instead of stating that they subtend equal angles at the centre.

How to avoid it: Remember the difference. A chord's length is a linear distance measured in centimetres, while a subtended angle is a rotational measurement expressed in degrees.

8. QUICK REVISION NOTES

- A circle is a collection of all points in a plane that are equidistant from a fixed centre point.
- The diameter is the longest chord of a circle and is exactly twice the length of the radius ($d = 2r$).
- Equal chords always subtend equal angles at the centre of the circle.
- A perpendicular line drawn from the centre to a chord always cuts that chord into two equal halves.
- Equal chords are always at an equal perpendicular distance from the centre of the circle.
- A sector is bounded by two radii and an arc, while a segment is bounded by a chord and an arc.

9. ONE-PAGE SUMMARY

This chapter outlines the core geometric theorems that govern chords, lines, and distances within a circle:

CIRCULAR COMPONENT	GOVERNING THEOREM / PROPERTY	CORE GEOMETRIC RELATIONSHIP
Equal Chords	Central Angle Property	Subtend equal angles at the centre ($\angle AOB = \angle COD$).
Perpendicular from Centre	Bisection Property	Cuts the chord into two equal halves ($AM = MB$).
Equal Chord Distances	Equidistance Property	Chords are located at equal distances from the center ($OM = ON$).
Diameter Path	Longest Chord Rule	Passes through the centre; length equals $2r$.

10. MIND MAP

Circles Geometry

- └─ 1. Boundary Foundations
 - └─ Component Metrics: Centre (O), Radius (r), Longest Chord / Diameter (2r)
 - └─ Boundary Perimeter: Circumference represents total linear length
- └─ 2. Internal Partitioning
 - └─ Sector: Block region bounded by 2 radii and 1 arc (Pizza wedge)
 - └─ Segment: Block region bounded by 1 chord and 1 arc (Bread slice)
- └─ 3. Core NCERT Theorems
 - └─ Angle Subtensions: Equal chords create equal central angles
 - └─ Perpendicular Lines: Drop from origin bisects a chord perpendicularly
 - └─ Distance Invariance: Equal chords remain equidistant from the centre

11. EXAM TIPS

- **Always Write Theorem Reasons in Brackets:** When calculating lengths or angles, **always** write the full theorem name or property in brackets next to your step (e.g., `[Perpendicular from centre to a chord bisects the chord]`). Leaving out these justifications is a frequent cause of lost marks.
- **Use Right Triangles to Find Unknown Lengths:** Most chord calculation questions can be solved by drawing a perpendicular line from the centre to the chord. This creates a right-angled triangle, allowing you to use Pythagoras Theorem ($\text{Radius}^2 = \text{Distance}^2 + (\text{Chord}/2)^2$) to find the answer.

12. QUICK FORMULA & THEOREM SHEET

- **Diameter Identity:** $d = 2r$
- **Pythagorean Chord Relation:** $OB^2 = OM^2 + MB^2$ (where OB = radius, OM = distance from centre, MB = half of chord length)
- **Equal Chords Angle Balance:** $AB = CD \Leftrightarrow \angle AOB = \angle COD$
- **Equidistance Equality Balance:** $AB = CD \Leftrightarrow OM = ON$ (where OM, ON are perpendicular distances)

Ultra-Fast Revision Box

CRITICAL THEOREM MEMORY BOOSTER

Remember that a perpendicular line drawn from the centre of a circle ****always bisects the chord****. This property links chord lengths to right-angled triangles, providing an easy way to solve most calculation problems using Pythagoras Theorem.

30-Second Exam Revision Section

- Points equidistant from a centre $\Rightarrow$ Forms a valid circle boundary
- Perpendicular line from centre $\Rightarrow$ Cuts the chord into two equal halves
- Equal chords $\Rightarrow$ Subtend equal angles at the centre and sit at equal distances from it
- SeCtor vs. SegMent $\Rightarrow$ Sectors connect to the Centre; Segments are bounded by a Middle chord